1. A particle of mass m is projected vertically upwards, at time $t = 0$, with speed U. The particle is subject to air resistance of magnitude $\frac{mgv^2}{k^2}$, where v is the speed of the particle at time t and k is a positive constant.

(a) Show that the particle reaches its greatest height above the point of projection at time $\frac{k}{g}\tan^{-1}\left(\frac{U}{k}\right)$.

(6)

(b) Find the greatest height above the point of projection attained by the particle.

(6)
(Total 12 marks)

2. A train of mass m is moving along a straight horizontal railway line. At time t, the train is moving with speed v and the resistance to motion has magnitude kv, where k is a constant. The engine of the train is working at a constant rate P.

(a) Show that, when $v > 0$, $$mv\frac{\mathrm{d}v}{\mathrm{d}t} + kv^2 = P.$$

(3)

When $t = 0$, the speed of the train is $\frac{1}{3}\sqrt{\left(\frac{P}{k}\right)}$.

(b) Find, in terms of m and k, the time taken for the train to double its initial speed.

(8)
(Total 11 marks)

3. A particle P of mass 0.5 kg is released from rest at time $t = 0$ and falls vertically through a liquid. The motion of P is resisted by a force of magnitude $2v$ N, where v m s^{-1} is the speed of P at time t seconds.

(a) Show that $5\frac{\mathrm{d}v}{\mathrm{d}t} = 49 - 20v$

(2)

(b) Find the speed of P when $t = 1$.

(5)
(Total 7 marks)

4. A lorry of mass M is moving along a straight horizontal road. The engine produces a constant driving force of magnitude F. The total resistance to motion is modelled as having magnitude kv^2, where k is a constant, and v is the speed of the lorry.

Given the lorry moves with constant speed V,

(a) show that $V = \sqrt{\frac{F}{k}}$.

(2)

Given instead that the lorry starts from rest,

(b) show that the distance travelled by the lorry in attaining a speed of $\frac{1}{2}V$ is

$$\frac{M}{2k}\ln\left(\frac{4}{3}\right).$$

(9)
(Total 11 marks)

5. A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity $2mak^2$, where k is a positive constant. The other end of the string is attached to a fixed point A. At time $t = 0$, P is released from rest from a point which is a distance $2a$ vertically below A. When P is moving with speed v, the air resistance has magnitude $2mkv$. At time t, the extension of the string is x.

(a) Show that, while the string is taut,

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + 2k^2x = g.$$

(4)

You are given that the general solution of this differential equation is

$$x = e^{-kt}(C \sin kt + D \cos kt) + \frac{g}{2k^2}, \quad \text{where } C \text{ and } D \text{ are constants.}$$

(b) Find the value of C and the value of D.

(5)

Assuming that the string remains taut,

(c) find the value of t when P first comes to rest,

(3)

(d) show that $2k^2a < g(1 + e^{\pi})$.

(4)
(Total 12 marks)

6. A particle P of mass m is attached to one end of a light inextensible string and hangs at rest at time $t = 0$. The other end of the string is then raised vertically by an engine which is working at a constant rate kmg, where $k > 0$. At time t, the distance of P above its initial position is x, and P is moving upwards with speed v.

(a) Show that $v^2\dfrac{dv}{dx} = (k - v)g$.

(4)

(b) Show that $gx = k^2 \ln\left(\frac{k}{k - v}\right) - kv - \frac{1}{2}v^2$.

(7)

(c) Hence, or otherwise, find t in terms of k, v and g.

(5)
(Total 16 marks)

7. A particle P of mass 3 kg moves in a straight line on a smooth horizontal plane. When the speed of P is v m s^{-1}, the resultant force acting on P is a resistance to motion of magnitude $2v$ N. Find the distance moved by P while slowing down from 5 m s^{-1} to 2 m s^{-1}.

(Total 5 marks)

8. A particle P of mass m is attached to the mid-point of a light elastic string, of natural length $2L$ and modulus of elasticity $2mk^2L$, where k is a positive constant. The ends of the string are attached to points A and B on a smooth horizontal surface, where $AB = 3L$. The particle is released from rest at the point C, where $AC = 2L$ and ACB is a straight line. During the subsequent motion P experiences air resistance of magnitude $2mkv$, where v is the speed of P. At time t, $AP = 1.5L + x$.

(a) Show that $\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + 4k^2x = 0$.

(6)

(b) Find an expression, in terms of t, k and L, for the distance AP at time t.

(8)
(Total 14 marks)

9. A wooden ball of mass 0.01 kg falls vertically into a pond of water. The speed of the ball as it enters the water is 10 m s^{-1}. When the ball is x metres below the surface of the water and moving downwards with speed v m s^{-1}, the water provides a resistance of magnitude $0.02v^2$ N and an upward buoyancy force of magnitude 0.158 N.

(a) Show that, while the ball is moving downwards,

$$-2v^2 - 6 = v\frac{dv}{dx}.$$

(3)

(b) Hence find, to 3 significant figures, the greatest distance below the surface of the water reached by the ball.

(5)
(Total 8 marks)

10. A small pebble of mass m is placed in a viscous liquid and sinks vertically from rest through the liquid. When the speed of the pebble is v the magnitude of the resistance due to the liquid is modelled as mkv^2, where k is a positive constant.

Find the speed of the pebble after it has fallen a distance D through the liquid.

(Total 11 marks)

$v^2 - 6 = v\frac{dv}{dx}$

11. A car of mass 1000 kg, moving along a straight horizontal road, is driven by an engine which produces a constant power of 12 kW. The only resistance to the motion of the car is air resistance of magnitude $10v^2$ N where v m s^{-1} is the speed of the car.

Find the distance travelled by the car as its speed increases from 5 m s^{-1} to 10 m s^{-1}.

(Total 8 marks)

(6)

(b) Find an expression, in terms of t, k and L, for the distance AP at time t.

(8)
(Total 14 marks)

12. A body falls vertically from rest and is subject to air resistance of a magnitude which is proportional to its speed.

Given that its terminal speed is 100 m s^{-1}, find the time it takes for the body to attain a speed of 60 m s^{-1}.

(Total 10 marks)

13. A particle P of mass m is fixed to one end of a light elastic string, of natural length a and modulus of elasticity $2man^2$. The other end of the string is attached to a fixed point O. The particle P is released from rest at a point which is a distance $2a$ vertically below O. The air resistance is modelled as having magnitude $2mnv$, where v is the speed of P.

(a) Show that, when the extension of the string is x,

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 2n\frac{\mathrm{d}x}{\mathrm{d}t} + 2n^2 x = g$$

(5)

(b) Find x in terms of t.

(7)
(Total 12 marks)

1. (a)

$$-mg\left(1+\frac{v^2}{k^2}\right)=m\frac{\mathrm{d}v}{\mathrm{d}t}$$ M1 A1

$$g\int_0^T \mathrm{d}t=\int_U^0\frac{-k^2\mathrm{d}v}{(k^2+v^2)}$$ DM1

$$T=\frac{k}{g}\left[\tan^{-1}\frac{v}{k})\right]_0^U$$ A1

$$=\frac{k}{g}\tan^{-1}\frac{U}{k}$$ DM1 A1 6

(b)

$$-mg\left(1+\frac{v^2}{k^2}\right)=mv\frac{\mathrm{d}v}{\mathrm{d}x}$$ M1 A1

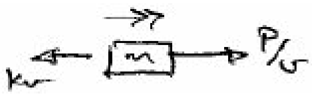
$$g\int_0^H \mathrm{d}x=\int_U^0\frac{-k^2v\mathrm{d}v}{(k^2+v^2)}$$ DM1

$$H=\frac{k^2}{2g}\left[\ln(k^2+v^2\right]_0^U$$ A1

$$H=\frac{k^2}{2g}\ln\frac{(k^2+U^2)}{k^2}$$ DM1 A1 6

[12]

2. (a)



$\frac{P}{v}$ B1

$$(\rightarrow):\frac{P}{v}-kv=m\frac{dv}{dt}$$ M1

$$\Rightarrow P=mv\frac{dv}{dt}+kv^2$$ A1 3

(b)

$$\int_?^T dt=\int_u^{2u}\frac{mvdv}{P-\mathrm{K}v^2}\qquad\left(u=\frac{1}{3}\sqrt{\frac{P}{\mathrm{K}}}\right)$$ M1 A1

$$\Rightarrow T=-\frac{m}{2\mathrm{K}}\left[\ln(P-\mathrm{K}v)\right]_u^{2u}$$ A2

$$=\frac{\mathrm{m}}{2\mathrm{k}}\{\ln(\mathrm{P}-\frac{\cancel{\mathrm{k}}}{9}\frac{\mathrm{p}}{\cancel{\mathrm{k}}})-\ln(\mathrm{P}-\frac{4\cancel{\mathrm{k}}}{9}\frac{\mathrm{p}}{\cancel{\mathrm{k}}})\}$$ M1 A1

$$=\frac{m}{2\mathrm{K}}\{\ln\frac{8P}{9}-\ln\frac{5P}{9}\}$$

$$=\frac{m}{2\mathrm{K}}\{\ln(\frac{8P}{9}\times\frac{9}{5P}\}$$ M1

$= \dfrac{m}{2K} \ln \dfrac{8}{5}$ A1 8

[11]

3. (a) $\dfrac{1}{2}\dfrac{dv}{dt} = \dfrac{1}{2}g - 2v$ M1

$\Rightarrow \underline{5\dfrac{dv}{dt} = 49 - 20v}$ A1 2

(b) $\displaystyle\int \frac{5dv}{49 - 20v} = \int dt$ (separate variables) M1

$\dfrac{-5}{20} 1n(49 - 20v) = t\ (+c)$ A1

$t = 0,\ v = 0 \Rightarrow c = -\ \dfrac{1}{4}\ \ln 49$ (attempt to get c) M1

$t = \dfrac{1}{4} 1n\left(\dfrac{49}{49 - 20v}\right)$

$t = 1 : 1 = \dfrac{1}{4} 1n\left(\dfrac{49}{49 - 20v}\right)$ (correct use of logs/exp) M1

$\rightarrow \underline{v \approx 2.41 ms^{-1} \quad \text{or} \quad 2.4 ms^{-1}}$ A1 5

[7]

4. (a) For constant speed, $F - kV^2 = 0$ M1

$\Rightarrow \underline{V = \sqrt{F/k}}$ (*) A1 2

(b) $F - kv^2 = Ma$ M1 A1

$\Rightarrow F - kv^2 = Mv\ \dfrac{dv}{dx}$ M1

$\displaystyle\int dx = M\int \frac{v}{F - kv^2}\, dv$ M1

$x = -\dfrac{M}{2k} \ln(F - kv^2)\ (+C)$ A1

$x = 0,\ v = 0 \Rightarrow C = \dfrac{M}{2k} \ln F$ M1 A1

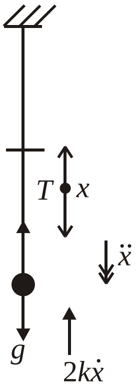
$x = \dfrac{M}{2k} \{\ln F - \ln(F - kv^2)\}$

$x = \dfrac{M}{2k} \ln\left(\dfrac{F}{F - k.\dfrac{F}{4k}}\right)$ M1

$= \underline{\dfrac{M}{2k}} \ln \underline{\dfrac{4}{3}}$ (*) A1 9

[11]

5. (a)



R(↓), $mg - 2k\dot{x}m - T = m\ddot{x}$

$g - 2k\dot{x} - \dfrac{2ak^2x}{a} = \ddot{x}$ M1

$\Rightarrow \underline{\ddot{x} + 2k\dot{x} + 2k^2x = g}$ (*) A1 c.s.o 4

(b) $t = 0, x = a$: $a = D + \dfrac{g}{2k^2} \Rightarrow \underline{D = a - \dfrac{g}{2k^2}}$ M1 A1

$\dot{x} = -ke^{-kt}(C \sin kt + D \cos kt) + -ke^{-kt}(C \cos kt - D \sin kt)$ M1 A1

$t = 0,\ \dot{x} = 0$: $0 = -kD + kC \Rightarrow C = D$

$\Rightarrow \underline{C = a} - \dfrac{g}{2k^2}$ A1 5

(c) $\dot{x} = 0$

$\Rightarrow C(\sin kt + \cos kt) + C(\cos kt - \sin kt)$ M1

$\Rightarrow \sin kt = 0$ A1 ft

$\Rightarrow kt = \pi$

$\Rightarrow \underline{t = \dfrac{\pi}{k}}$ A1 3

(d) When $t = \dfrac{\pi}{k}$, $x = -De^{-\pi} + \dfrac{g}{2k^2}$ M1

$\dfrac{g}{2k^2} - e^{-\pi}\left(a - \dfrac{g}{2k^2}\right)$ A1 ft

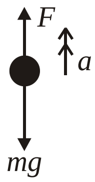
$\Rightarrow xe^{\pi} = \dfrac{g}{2k^2}(e^{\pi} + 1) - a$

> 0 (given) M1

$\Rightarrow g(e^{\pi} + 1) > 2k^2a$ (*) A1 c.s.o 4

[16]

6. (a)



$F = \dfrac{kmg}{v}$ B1

R(↑), $F - mg = ma$ M1

$\dfrac{kmg}{v} - mg = mv\dfrac{dv}{dx}$ M1

$\underline{g(k - v) = v^2\dfrac{dv}{dx}}$ (*) A1 4

(b) $g\,dx = \dfrac{v^2}{k - v}\,dv$ M1

$\int g\,dx = \int -v - k + \dfrac{k^2}{k - v}\,dv$ M1 A1

$gx = -\dfrac{v^2}{2} - kv - k^2\ln(k - v) + (c)$ M1 A1

$x = 0,\ v = 0$

$0 = 0 - 0 - k^2\ln k + c$ M1

$c = k^2\ln k$

$\underline{gx = -\dfrac{v^2}{2} - kv - k^2\ln\left(\dfrac{k}{k - v}\right)}$ (*) A1 7

(c) Work done by engine = Energy gain

$kmgt = \dfrac{1}{2}mv^2 + mgx$ M1 A1

$kmgt = mk^2\ln\left(\dfrac{k}{k - v}\right) - mkv$ M1

$\Rightarrow \underline{t = \dfrac{k}{g}\ln\left(\dfrac{k}{k - v}\right) - \dfrac{v}{g}}$ A1 5

Alternative:

$G(k - v) = v\dfrac{dv}{dt}$

$\Rightarrow dt = \dfrac{v\,dv}{g(k - v)}$ M1

$\Rightarrow dt = \dfrac{1}{g}\left(-1 + \dfrac{k}{k - v}\right)$ A1

$\Rightarrow t = \dfrac{1}{g}(-v - k\ln(k - v) + c$ A1

$t = 0,\ v = 0 \Rightarrow c = k\ln\dfrac{k}{g}$

$\Rightarrow t = -\frac{v}{g} + \frac{k}{g}(\ln k - \ln(k - v))$ M1

i.e. $t = -\frac{v}{g} + \frac{k}{g}\ln\left(\frac{k}{k-v}\right)$ A1 (5)

[16]

7. N2L $-2v = 3a$ M1

$-2v = 3v\frac{\mathrm{d}v}{\mathrm{d}s}$ A1

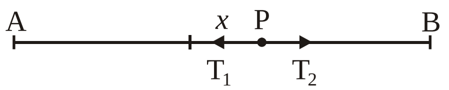
$s = -\frac{3}{2}v(+c)$ or $v = -\frac{2}{3}s\ (+c)$ **cancelling *v* and integrating** M1

$s = 0,\ v = 5 \Rightarrow c = \frac{15}{2}$ or $s = \left[-\frac{3}{2}v\right]_5^2$ M1

Distance travelled is 4.5 m A1 5

[5]

8. (a)



HL $T_1 = \frac{2mk^2L(0.5L + x)}{L}$ either M1

HL $T_2 = \frac{2mk^2L(0.5L - x)}{L}$ both A1

N2L $T_2 - T_1 - 2mk\frac{\mathrm{d}x}{\mathrm{d}t}, = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ M1 A1, A1

$4mk^2x - 2mk\frac{\mathrm{d}x}{\mathrm{d}t} = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$

$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2k\frac{\mathrm{d}x}{\mathrm{d}t} + 4k^2x = 0$ * cso A1 6

(b) $m^2 + 2km + 4m^2 = 0$ ae M1

$m = -k \pm k\sqrt{3}\mathrm{i}$ M1

$x = \mathrm{e}^{-kt}(A\cos\sqrt{3}kt + B\sin\sqrt{3}kt)$ oe A1

$t = 0,\ x = \frac{L}{2} \Rightarrow A = \frac{L}{2}$ B1

$\dot{x} = -k\mathrm{e}^{-kt}(A\cos\sqrt{3}kt + B\sin\sqrt{3}kt) + \sqrt{3}k\mathrm{e}^{-kt}(-A\sin\sqrt{3}kt + B\cos\sqrt{3}kt)$ M1

$t = 0,\ \dot{x} = 0 \Rightarrow 0 = -kA + \sqrt{3}kB$ M1

$B = \frac{1}{\sqrt{3}}A = \frac{L}{2\sqrt{3}}$ A1

$AP = 1.5L + \frac{L}{2\sqrt{3}}\mathrm{e}^{-kt}(\sqrt{3}\cos\sqrt{3}kt + \sin\sqrt{3}kt)$ oe A1 8

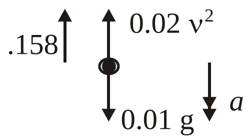
Alternative form of the General Solution As before M1 M1

$X = A\,e^{-kt}\cos(\sqrt{3}kt - \varepsilon)$	A1	
$t = 0,\ x = \dfrac{L}{2} \Rightarrow \dfrac{L}{2} = A\cos(-\varepsilon)\ (= A\cos\varepsilon)$	B1	
$\dot{x} = -kAe^{-kt}\cos(\sqrt{3}kt - \varepsilon) - \sqrt{3}kA\,e^{-kt}\sin(\sqrt{3}kt - \varepsilon)$	M1	
$t = 0,\ \dot{x} = 0 \Rightarrow 0 = -kA\cos\varepsilon - \sqrt{3}kA\sin(-\varepsilon)$	M1	
Leading to $\tan\varepsilon = \dfrac{1}{\sqrt{3}} \Rightarrow \varepsilon = \dfrac{\pi}{6}$ and $A = \dfrac{L}{\sqrt{3}}$	both A1	
$AP = 1.5L + \dfrac{L}{\sqrt{3}}e^{-kt}\cos\left(\sqrt{3}kt - \dfrac{\pi}{6}\right)$	A1	8

Note: Another possible trig form is $\sin\left(\sqrt{3}kt + \dfrac{\pi}{3}\right)$

[14]

9. (a)



$0.01a = 0.01g - 0.158 - 0.02v^2$	M1	
$a = v\dfrac{dv}{dx}$	M1	
$v\dfrac{dv}{dx} = -2v^2 - 6$ (*)	A1	3

(b)

$-\displaystyle\int \frac{v\,dv}{2v^2+6} = \int dx$	M1	
$x = -\frac{1}{4}\ln(2v^2 + 6)\ (+ C)$	A1 ft $\left(\frac{1}{4}\right)$	
$x = 0,\ v = 0 \Rightarrow C = \frac{1}{4}\ln 206$	A1	
$v = 0 \Rightarrow x = \frac{1}{4}\ln\dfrac{206}{6} \approx 0.884$ m	M1 A1	5

[8]

10.

$(\downarrow)\ mg - mkv^2 = ma$	M1 A1	
$g - kv^2 = v\dfrac{dv}{dx}\quad v\dfrac{dv}{dx}$	M1	
$x = \displaystyle\int \frac{v}{g - kv^2}\,dv$	M1	
$x = -\dfrac{1}{2k}\ln\lvert g - kv^2\rvert + c$	M1 A1	
$x = 0,\ v = 0 \Rightarrow 0 = -\dfrac{1}{2k} + c$	M1	

$x = \frac{1}{2k}\ln\left|\frac{g}{g - kv^2}\right|$

$e^{2kx} = \frac{g}{g - kv^2}$ A1

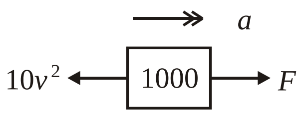
$kv^2 = g(1 - e^{-2kx})$ M1

$v = \sqrt{\frac{g}{k}\left(1 - e^{-2kD}\right)}$ A1

must use D

[11]

11.



$F = \frac{12000}{v}$ B1

$\frac{12000}{v} - 10v^2 = 1000v\,\frac{dv}{dx}$ M1 A1

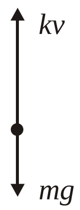
$\int dx = 100\int \frac{v^2\,dv}{1200 - v^3}$ M1

$X = -\frac{100}{3}\left[\ln(1200 - v^3)\right]_5^{10}$ M1 A1

$= 56.1$ m (3 s.f.) M1 A1

[8]

12.



$mg - 100k = 0$ at terminal speed M1

$k = \frac{mg}{100}$ A1

$mg - \frac{mg}{100}v \;.= \; m\frac{dv}{dt}$ M1 A1 A1

$\int dt = \frac{100}{g}\int \frac{dv}{100 - v}$ M1

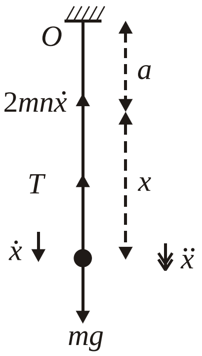
$T = \frac{100}{g}\left[\ln(100 - v)\right]_{60}^{0}$ A1 A1(limits)

$= \dfrac{100}{g} \ln\left(\dfrac{100}{40}\right)$ M1

$= 9.35$ s (3 s.f.) A1 10

[10]

13. (a)



$mg - T - 2mn\dot{x} = m\ddot{x}$ M1 A1 A1

$mg - \dfrac{2man^2 x}{a} - 2mn\dot{x} = m\ddot{x}$ M1

$\ddot{x} + 2n\dot{x} + 2n^2 x = g$ (*) A1 5

(b) AE: $u^2 + 2nu + 2n^2 = 0$ M1

$(u + n)^2 = -n^2$

$u = -n \pm n\mathrm{i}$ A1

CF: $x = e^{-nt}(A \cos nt + B \sin nt)$, PI: $x = \dfrac{g}{2n^2}$ M1

GS: $x = e^{-nt}(A \cos nt + B \sin nt) + \dfrac{g}{2n^2}$ A1

$t = 0, x = a, \dot{x} = 0$: $A = a - \dfrac{g}{2n^2}$ M1

$\dot{x} = e^{-nt}(-An \sin nt + Bn \cos nt) - ne^{-nt}(A \cos nt + B \sin nt)$ M1

$x = e^{-nt}\left(a - \dfrac{g}{2n^2}\right)(\cos nt + \sin nt) + \dfrac{g}{2n^2}$ A1 7

[12]

1. This question followed a standard format and was answered very well by a large proportion of candidates. A majority took the constant of integration approach rather than the use of definite integration, but generally did so successfully in both parts (a) and (b). A small number of candidates were confused about the directions involved and often tried to justify a change of sign part way through their working in order to arrive at the given answer in part (a).

A small number of candidates attempted to use their answer to part (a) to generate a differential equation in $\frac{dx}{dt}$ for part (b) rather than the simpler method of restarting from the beginning and using $v\frac{dv}{dx}$. These candidates were rarely able to obtain an expression for v correctly, or to integrate their expression successfully, and tended to grind to a halt with little progress made.

2. (a) Many candidates were not very convincing in their application of Newton's second law to produce the required equation. The symbol F frequently had two different meanings within the solution; it would be very much clearer if teachers were to encourage their pupils to use a different symbol such as D for the driving force.

(b) The differential equation was usually solved successfully, although some weaker candidates were unable to separate the variables. A few candidates tried to use an integrating factor, but often failed to realise that v^2 should be used rather than v for that method to work.

3. This was a good starter question with many fully correct solutions. A few candidates made an error in (a) and then used their incorrect answer instead of the printed answer in (b). The integration and manipulation of exponential and log functions was good.

4. This was a standard problem on which most candidates scored well. Common errors involved incorrect signs in the integration or the omission of a factor of k. The constant of integration was not included in some cases but on the whole the question was well done.

5. Those who had the particle moving upwards in (a) made setting up the equation much more difficult – it is always best, when considering a general position, to assume that the particle is moving in the positive x-direction. Basic errors were often made in (b), either use of incorrect boundary conditions or else incorrect differentiation and despite the hint in (c), only the best were able to answer the final part.

6. Most candidates managed to obtain the printed answer in part (a), although a few had to resort to fiddling their working. In the second part, the integration caused a lot of problems - those who tried using integration by parts got nowhere but the last part was usually done more successfully – almost all of the candidates set up a new differential equation, rather than use a hence method, and were able to deal with the easier integral.

7. This proved an accessible start to the paper although a few candidates had an incorrect sign in their equation of motion. In carrying out the integration a number of candidates had the limits in the wrong order, integrating from $t = 2$ to $t = 5$, and dropped the resulting negative sign without further explanation.

8. Part (a) proved difficult for many candidates. Although nearly all obtained the printed expression one way or another, the working was often incorrect. The commonest mistake was to have incorrect expressions for the extensions of *AP* and *BP*, which should have been $0.5L+x$ and $0.5L–x$ respectively. A minority had the wrong number of terms in their equation of motion and some of these clearly failed to appreciate that both portions of the string were under tension.

The pure mathematical manipulation in part (b) is quite demanding and it is pleasing to report that the majority of candidates displayed excellent technical skills in dealing with this part of the question. Many candidates lost the last mark in the question, failing to recognise that as they were asked to find *AP*, $1.5L$ had to be added to their expression for x.

9. This proved to be a friendly starter for the vast majority of candidates with several fully correct solutions seen. There were some arithmetical slips in the integration in part (b), but generally candidates knew what they were meant to do here.

10. This question was answered well and most candidates demonstrated a sound knowledge of this topic. The standard of integration, application of boundary conditions and algebraic processing was high and many students scored full marks here. A few failed to use $v\frac{dv}{dx}$ for the acceleration and some did not include a constant of integration and hence were unable to apply the boundary conditions.

11. No Report available for this question.

12. No Report available for this question.

13. No Report available for this question.